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Modular Bus Scheduling Problem with Short-turning Options

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Research Contribution

This research contributes to the **state of the art** by allowing for the **simultaneous** deployment of **modular buses and short-turning options**, which have been previously studied separately.

The use of modular buses for short-turning **can lead to a more efficient resource utilization compared to conventional fixed-line bus services.**
To investigate this potential benefit, our study introduces **a mixed-integer non-linear program** that determines **the number of modular units** and possible short-turn options for all bus trips on a bus line, which provides significant value for **bus operators**.

Study Contributions:

- Develops, for the first time, a **unifying mathematical formulation** for the scheduling of autonomous modular buses considering short-turning options.
- **Linearizes** constraints of the developed mathematical model to **reduce its complexity**.
- Highlights the potential **improvement** compared to the **as-is operations** performed by conventional buses.
- Demonstrates the potential impact of the proposed modular bus scheduling model with short-turning options using data from a **real case study** of an urban bus service in **Milan, Italy**.

Modular Bus Scheduling Problem with Short-turning Options

$$\min \sum_{t\in T}\sum_{s\in S}\sum_{u\in U} x_{s,ut}c_{s,u,t}$$

s.t.

$$\sum_{u\in U}\sum_{S\in S} x_{s,u,t} = 1, \qquad \forall t\in T \qquad (1)$$

$$\widetilde{b_{t,k}} \doteq \sum_{t\in T} w_{t,k,y} \sum_{u\in U}\sum_{s\in S} x_{s,u,t}\,\beta_{s,k,y} \qquad \forall t\in T, k\in K \qquad (2)$$

$$l_{1,1} \le \gamma \sum_{u\in U}\sum_{S\in S} x_{s,u,1}u \qquad (3)$$

$$l_{1,1} \le \widetilde{b_{1,1}} \qquad (4)$$

$$l_{1,1} \ge \gamma \sum_{u\in U}\sum_{S\in S} x_{s,u,1}U - d_{1,1}M \qquad (5)$$

$$l_{1,1} \ge \widetilde{b_{1,1}} + (d_{1,1}-1)M \qquad (6)$$

$$a_{1,1} \doteq \sum_{y\in K, y\ge 1} w_{1,1,y} - l_{1,1} \qquad (7)$$

$$z_{1,1} \doteq \frac{l_{1,1}}{\sum_{y\in K,y\ge 1} w_{1,1,y}} \qquad (8)$$

$$g_{1,1,y} = (1-z_{1,1})w_{1,1,y}, \qquad \forall y\in K, y\ge 1 \qquad (9)$$

$$l_{1,k} \le l_{1,k-1} + \widetilde{b_{1,k}} - \sum_{y\in K,y<k} z_{1,y}\, w_{1,y,k} \sum_{S\in S}\sum_{u\in U} x_{s,u,1}\,\beta_{s,y,k}, \forall k\in K\setminus\{1\}, \forall y\in K, y<k \qquad (10)$$

$$l_{1,k} \le \gamma \sum_{S\in S}\sum_{u\in U} x_{s,u,1}\,U, \qquad \forall k\in K\setminus\{1\} \qquad (11)$$

$$l_{1,k} \ge \mathrm{l}_{1,k-1} + \widetilde{b_{1,k}} - \sum_{y\in K,y<k} z_{1,y}\, w_{1,y,k} \sum_{s\in S}\sum_{u\in U} x_{s,u,1}\;\beta_{s,y,k}\,(d_{1,k}-1)M \approx \forall k\in K\setminus\{1\} \qquad (12)$$

$$l_{1,k} \ge \gamma \sum_{u\in U}\sum_{S\in S} x_{s,u,1}U - (d_{t,k})M, \qquad \forall k\in K\setminus\{1\} \qquad (13)$$

$$z_{1,k} = \frac{l_{t,k} - l_{t,k-1} + \sum_{y\in K,y<k} z_{t,k}\, w_{t,k,y} \sum_{u\in U}\sum_{S\in S} x_{s,u,t}\beta_{s,y,k}}{\sum_{y\in K,y\ge k} w_{t,k,y}} \qquad \forall k\in K\setminus\{1\} \qquad (14)$$

$$a_{(1,k)} = \sum_{y\in K,y\ge k} w_{1,k,y} - (1-z_{1,k}) \qquad \forall k\in K\setminus\{1\} \qquad (15)$$

$$g_{1,k,y} = (1-z_{1,k})w_{1,k,y}, \qquad \forall k\in K, \forall y\in K: y\ge 1 \qquad (16)$$

$$\sum_{u\in U} x_{1,u,t} \doteq 1 \qquad (17)$$

$$l_{t,1} \le \gamma \sum_{u\in U}\sum_{S\in S} x_{1,u,t}u \qquad \forall t\in\{2,4,6,\dots\} \qquad (18)$$

$$l_{t,1} \doteq \sum_{y\in K,y\ge 1} w_{t,1,y} + \sum_{y\in K,y\ge 1} g_{t-1,1,y} \qquad \forall t\in\{2,4,6,\dots\} \qquad (19)$$

$$a_{t,1} \doteq a_{t-1,1} - l_{t,1} + \sum_{y\in K,y\ge 1} w_{t,1,y} \qquad \forall t\in\{2,4,6,\dots\} \qquad (20)$$

$$z_{t,1}\left(a_{t-1,1} + \sum_{y\in K,y\ge 1} w_{t,1,y}\right) = l_{t,1} \qquad \forall y\in\{2,4,6,\dots\} \qquad (21)$$

$$g_{t,1,y} \doteq (1-z_{t,1})w_{t,1,y}, \qquad \forall y\in K\sim|\sim y\ge 1 \forall t\in\{2,4,6,\dots\} \qquad (22)$$

Linearizations of the First Trip at the First Stop

These inequality constraints are linearizations of the following non-convex equality constraint:

$$l_{1,1} = \min\{\gamma \sum_{u\in U}\sum_{s\in S} x_{s,u,1}u\,;\, \widetilde{b_{1,1}}\}$$

s.t.:

$$l_{1,1} \le \gamma \sum_{u\in U}\sum_{S\in S} x_{s,u,1}u \qquad (3)$$

$$l_{1,1} \le \widetilde{b_{1,1}} \qquad (4)$$

$$l_{1,1} \ge \gamma \sum_{u\in U}\sum_{S\in S} x_{s,u,1}U - d_{1,1}M \qquad (5)$$

$$l_{1,1} \ge \widetilde{b_{1,1}} + (d_{1,1}-1)M \qquad (6)$$

Implementation & Application

The case study analyzes Line 54, which operates in **Milan, Italy**. The model has been implemented with Python, using the commercial solver Gurobi.

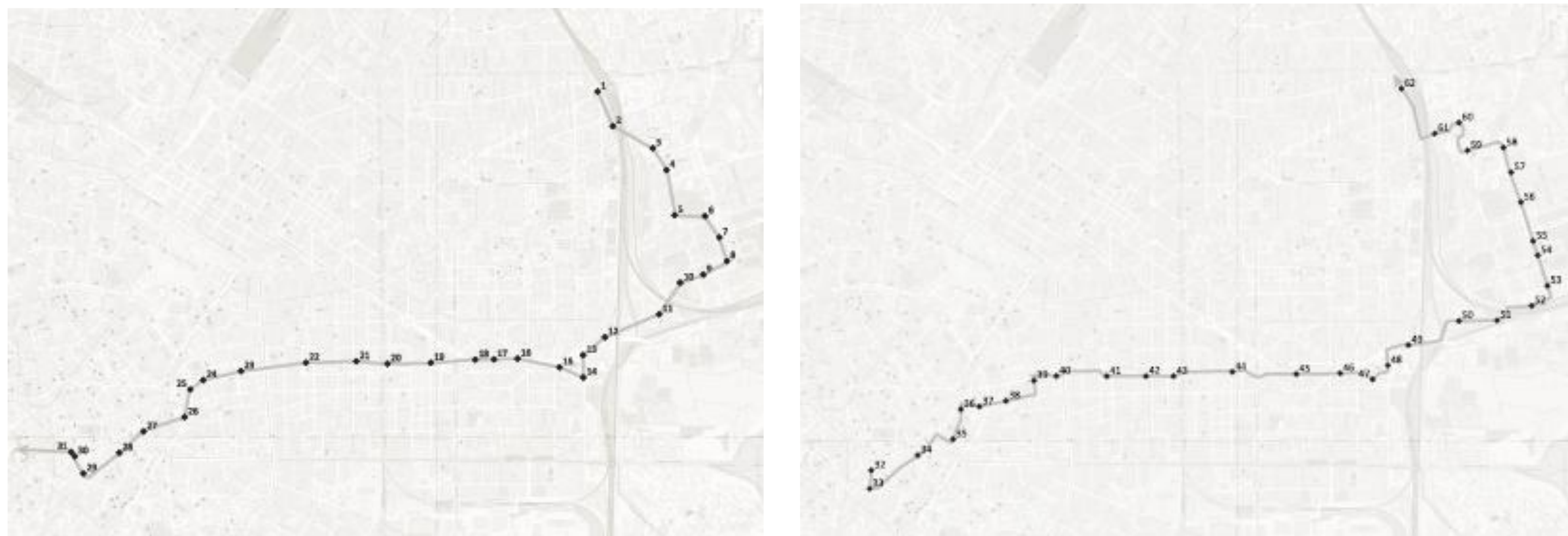

Figure 1. Topology of Milan's Line 54.

To visualize the difference between the proposed approach and the as-is scenario, Figure 2 presents **variations** in passenger loads when implementing each approach for trips 1 and 13, which had the highest passenger load variation.

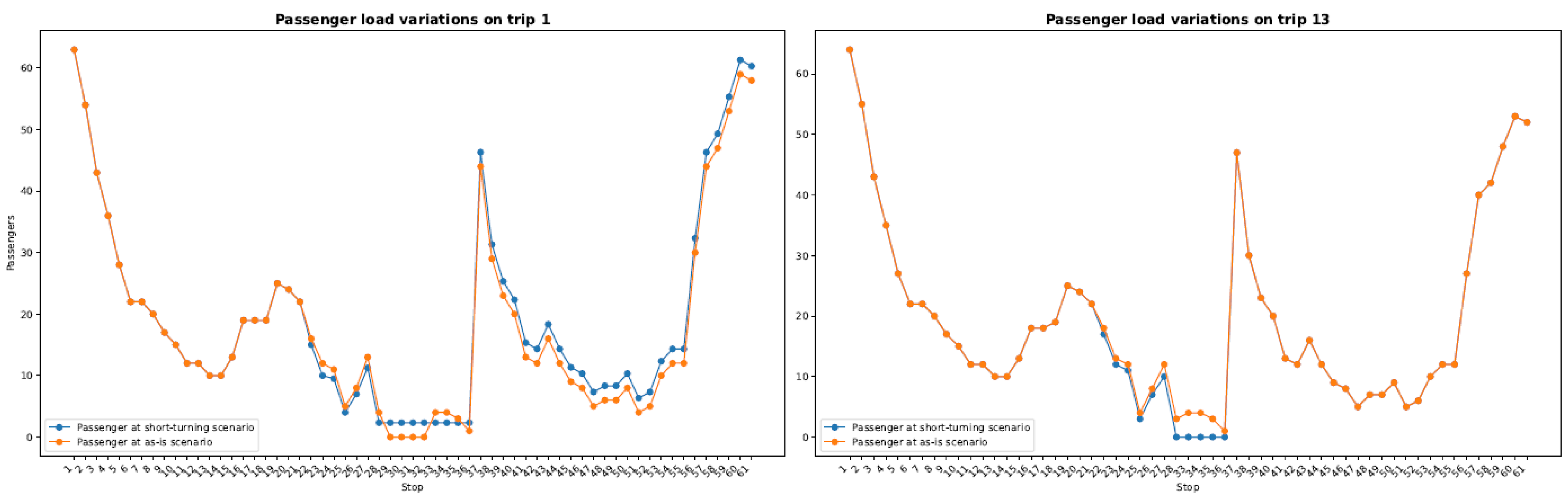

Figure 2. Passengers' load variations for two selected trips.

Trips	Modular bus scheduling with short-turning solution					As-is scenario				
	x^*	No of Skipped stops	No of modular units	km-traveled	Operational costs	x^*	No of modular units	km-traveled	Operational costs	Impro-vement
1	$x_{4,1,1}$	6	1	11.98	11.98	$x_{1,2,1}$	2	14.44	28.88	
2	$x_{1,1,2}$	2	1	12.55	12.55	$x_{1,2,2}$	2	14.44	28.88	
3	$x_{1,1,3}$	0	1	14.44	14.44	$x_{1,2,3}$	2	14.44	28.88	
4	$x_{1,1,4}$	0	1	14.44	14.44	$x_{1,2,4}$	2	14.44	28.88	
5	$x_{2,1,5}$	2	1	12.55	12.55	$x_{1,2,5}$	2	14.44	28.88	
6	$x_{1,1,6}$	0	1	14.44	14.44	$x_{1,2,6}$	2	14.44	28.88	
7	$x_{1,1,7}$	0	1	14.44	14.44	$x_{1,2,7}$	2	14.44	28.88	
8	$x_{1,1,8}$	0	1	14.44	14.44	$x_{1,2,8}$	2	14.44	28.88	
9	$x_{2,1,9}$	2	1	12.55	12.55	$x_{1,2,9}$	2	14.44	28.88	
10	$x_{1,1,10}$	0	1	14.44	14.44	$x_{1,2,10}$	2	14.44	28.88	
11	$x_{1,1,11}$	0	1	14.44	14.44	$x_{1,2,11}$	2	14.44	28.88	
12	$x_{1,1,12}$	0	1	14.44	14.44	$x_{1,2,12}$	2	14.44	28.88	
13	$x_{4,1,13}$	6	1	11.98	11.98	$x_{1,2,13}$	2	14.44	28.88	
14	$x_{1,1,14}$	0	1	14.44	14.44	$x_{1,2,14}$	2	14.44	28.88	
15	$x_{2,1,15}$	2	1	12.55	12,55	$x_{1,2,15}$	2	14.44	28.88	
16	$x_{1,1,16}$	0	1	14.44	14.44	$x_{1,2,16}$	2	14.44	28.88	
17	$x_{2,1,17}$	2	1	12.55	12.55	$x_{1,2,17}$	2	14.44	28.88	
18	$x_{1,1,18}$	0	1	14.44	14.44	$x_{1,2,18}$	2	14.44	28.88	
19	$x_{1,1,19}$	0	1	14.44	14.44	$x_{1,2,19}$	2	14.44	28.88	
20	$x_{1,1,20}$	0	1	14.44	14.44	$x_{1,2,20}$	2	14.44	28.88	
Total operational costs					274.43				577.6	52%
Total km-traveled				274.43				288.2		5%
Total modular units			20				40			50%

Key Findings

1. The model **has been tested for up to 20 bus trips** and can support a larger network
2. Comparing our approach with the as-is scenario **reveals variations in passenger loads**, with Trips 1 and 13 **showing the greatest differences**.

- In our case study of bus line **54 in Milan**, we demonstrated an improvement potential **of 50%** in terms of the number of deployed modular units, **5%** in terms of km-traveled, and **52%** in terms of overall operational costs.

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