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An Exact Bi-Level Optimization Model for the Charging Station Location Problem for Electric Buses

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Research Contribution

This study proposes an **exact bi-level optimization model** for the Charging Station Location Problem (CSLP) in **electric bus networks**.

The model attempts to capture the conflicting objectives of the **electric bus fleet operator** and the **energy management authority**, providing a coordinated and realistic planning framework that considers the perspective of both stakeholders.

Key features of our contribution:

- We formulate **a bi-level model** with an **upper-level MILP** for charger placement and charging scheduling, and **a lower-level LP** for tariff-setting by the energy authority.
- The **KKT conditions for the lower-level are calculated** and added to the upper-level, yielding a single-level Mathematical Program with Complementarity Constraints (MPCC) that can be solved to global optimality.
- The bi-level model includes parameters for: multiple charger types, real depot locations, disaggregated charging time slots, and detailed energy-transfer calculations.
- **Renewable energy participation** and **pricing deviations** are explicitly modeled, linking fleet operations to grid-management concerns.
- **A nested heuristic algorithm** is also introduced, offering near-optimal solutions.
- A real-world case study in **Limassol, Cyprus** demonstrates, better renewable energy utilization, and practical charger deployment strategies.

Electric Bus Operator Problem (Upper-level Problem Formulation)

$$\boldsymbol{Minimize\ O_1 = ds \cdot \left(DHC + TOUC^s + TOUC^h\right) + CSI} \quad (1)$$

s.t.:

$$\sum_{j\epsilon N} a_{kj}\, x_j \geq 1 \qquad \forall\, k \in K \quad (2)$$

$$q_{kj} \leq x_j \qquad \forall\, k \in K, j \in N \quad (3)$$

$$\sum_{k\epsilon K} q_{kj} \geq x_j \qquad \forall\, j \in N \quad (4)$$

$$\sum_{j\epsilon N} q_{kj} = 1 \qquad \forall\, k \in K \quad (5)$$

$$\sum_{f_1\epsilon F_1} u^s_{kjf_1} = q_{kj} \qquad \forall\, k \in K, j \in\ N_1 \quad (6)$$

$$\sum_{f_1\epsilon F_1} u^s_{kjf_1} = q_{kj} \qquad \forall\, k \in K, j \in\ N_1 \quad (7)$$

$$\sum_{k\epsilon K} u^s_{kjf_1} \leq 1 \qquad \forall\, j \in N_1, \forall\, f_1 \in F_1 \quad (8)$$

$$\sum_{k\epsilon K} u^h_{kjf_2} \leq 1 \qquad \forall\, j \in N_2, \forall\, f_2 \in F_2 \quad (9)$$

$$(1 - u^s_{kjf_1})M + u^s_{kjf_1} c^s_{f_1} \geq \left(\tau_k + t_{kj}\right) \qquad \forall\, k \in K, \forall\, j \in N_1, \forall\, f_1 \in F_1 \quad (10)$$

$$(1 - u^h_{kjf_2})M + u^h_{kjf_2} c^h_{f_2} \geq \left(\tau_k + t_{kj}\right) \qquad \forall\, k \in K, \forall\, j \in N_2, \forall\, f_2 \in F_2 \quad (11)$$

$$-(1 - u^s_{kjf_1})M + u^s_{kjf_1} c^s_{f_1} \geq \left(p^s_k + t_{kj}\right) \qquad \forall\, k \in K, \forall\, j \in N_1, \forall\, f_1 \in F_1 \quad (12)$$

$$-(1 - u^h_{kjf_2})M + u^h_{kjf_2} c^h_{f_2} \geq \left(p^h_k + t_{kj}\right) \qquad \forall\, k \in K, \forall\, j \in N_2, \forall\, f_2 \in F_2 \quad (13)$$

$$EC^s_{kjf_1} \leq M \cdot u^s_{kjf_1} \qquad \forall\, k \in K, \forall\, j \in N_1, \forall\, f_1 \in F_1 \quad (14)$$

$$EC^h_{kjf_2} \leq M \cdot u^h_{kjf_2} \qquad \forall\, k \in K, \forall\, j \in N_2, \forall\, f_2 \in F_2 \quad (15)$$

$$EC^s_{kjf_1} \geq\ (-M)\left(1\ -\ u^s_{kjf_1}\right) +\ PEC^s_{kj} \qquad \forall\, k \in K, \forall\, j \in N_1, \forall\, f_1 \in F_1 \quad (16)$$

$$EC^s_{kjf_1} \leq M\left(1\ -\ u^s_{kjf_1}\right) +\ PEC^s_{kj} \qquad \forall\, k \in K, \forall\, j \in N_1, \forall\, f_1 \in F_1 \quad (17)$$

$$EC^h_{kjf_2} \geq\ (-M)\left(1\ -\ u^h_{kjf_2}\right) +\ PEC^h_{kj} \qquad \forall\, k \in K, \forall\, j \in N_2, \forall\, f_2 \in F_2 \quad (18)$$

$$EC^h_{kjf_2} \leq M\left(1\ -\ u^h_{kjf_2}\right) +\ PEC^h_{kj} \qquad \forall\, k \in K, \forall\, j \in N_2, \forall\, f_2 \in F_2 \quad (19)$$

$$DC^s_{kjf_3} \leq M \cdot EC^s_{kjz^s_{f_3}} \qquad \forall\, k \in K, \forall\, j \in N_1, \forall\, f_1 \in F_1 \quad (20)$$

$$DC^h_{kjf_3} \leq M \cdot EC^h_{kjz^h_{f_3}} \qquad \forall\, k \in K, \forall\, j \in N_2, \forall\, f_2 \in F_2 \quad (21)$$

$$DC^s_{kjf_3} \geq\ (-M) \cdot u^s_{kjz^s_{f_3}} +\ DCP^s \qquad \forall\, k \in K, \forall\, j \in N_1, \forall\, f_3 \in F_3 : f_3 \leq FCS_{kjz^s_{f_3}} \quad (22)$$

$$DC^s_{kjf_3} \leq\ DCP^s +\ M \cdot u^s_{kjz^s_{f_3}} \qquad \forall\, k \in K, \forall\, j \in N_1, \forall\, f_3 \in F_3 : f_3 \leq FCS_{kjz^s_{f_3}} \quad (23)$$

$$DC^s_{kjf_3} \geq\ (-M) \cdot u^s_{kjz^s_{f_3}} +\ RE^s_{kj} \qquad \forall\, k \in K, \forall\, j \in N_1, \forall\, f_3 \in F_3 : f_3 = FCS_{kjz^s_{f_3}} + 1 \quad (24)$$

$$DC^s_{kjf_3} \leq\ RE^s_{kj} +\ M \cdot u^s_{kjz^s_{f_3}} \qquad \forall\, k \in K, \forall\, j \in N_1, \forall\, f_3 \in F_3 : f_3 = FCS_{kjz^s_{f_3}} + 1 \quad (25)$$

$$DC^h_{kjf_3} \geq\ (-M) \cdot u^h_{kjz^h_{f_3}} +\ DCP^h \qquad \forall\, k \in K, \forall\, j \in N_2, \forall\, f_3 \in F_3 : f_3 \leq FCS_{kjz^h_{f_3}} \quad (26)$$

$$DC^h_{kjf_3} \leq\ DCP^h +\ M \cdot u^h_{kjz^h_{f_3}} \qquad \forall k \in K, \forall\, j \in N_2, \forall\, f_3 \in F_3 : f_3 \leq FCS_{kjz^h_{f_3}} \quad (27)$$

$$DC^h_{kjf_3} \geq\ (-M) \cdot u^h_{kjz^h_{f_3}} +\ RE^h_{kj} \qquad \forall\, k \in K, \forall\, j \in N_2, \forall\, f_3 \in F_3 : f_3 = FCS_{kjz^h_{f_3}+1} \quad (28)$$

$$DC^h_{kjf_3} \leq\ RE^h_{kj} +\ M \cdot u^h_{kjz^h_{f_3}} \qquad \forall k \in K, \forall\, j \in N_2, \forall\, f_3 \in F_3 : f_3 = FCS_{kjz^h_{f_3}} + 1 \quad (29)$$

$$DEC_{f_3} = \textstyle\sum_{k\in K} \sum_{j\in N_1} DC^s_{kjf_3} + \sum_{k\in K} \sum_{j\in N_2} DC^h_{kjf_3} \qquad \forall\, f_3 \in F_3 \quad (30)$$

Energy Management Authority Problem (Lower-Level Formulation)

$$\boldsymbol{Maximize\ \ O_2 = \sum_{f_3\in F_3} RPP_{f_3} \cdot EARev_{f_3} - \sum_{f_3\epsilon F_3} PCP \cdot PC_{f_3}} \quad (31)$$

s.t.:

$$EARev_{f_3} = TOU_{f_3} \cdot DEC_{f_3} \qquad \forall\, f_3 \epsilon F_3 \quad (32)$$

$$EARev = \sum_{f_3\in F_3} EARev_{f_3} \quad (33)$$

$$EARev \geq EARev_{min} \quad (34)$$

$$PC_{f_3} \geq TOU_{f_3} - \widehat{TOU}_{f_3} \qquad \forall\, f_3 \epsilon F_3 \quad (35)$$

$$PC_{f_3} \geq \widehat{TOU}_{f_3} - TOU_{f_3} \qquad \forall\, f_3 \epsilon F_3 \quad (36)$$

$$PC_{f_3} \leq DEC_{f_3} \qquad \forall\, f_3 \epsilon F_3 \quad (37)$$

Implementation & Application

The bi-level has been implemented with Python 3.13.9, Gurobi Optimizer 11.0, Julia and BiLevelJuMP.jl library. The model has been applied to a real-world bus network from **Limassol, Cyprus**.

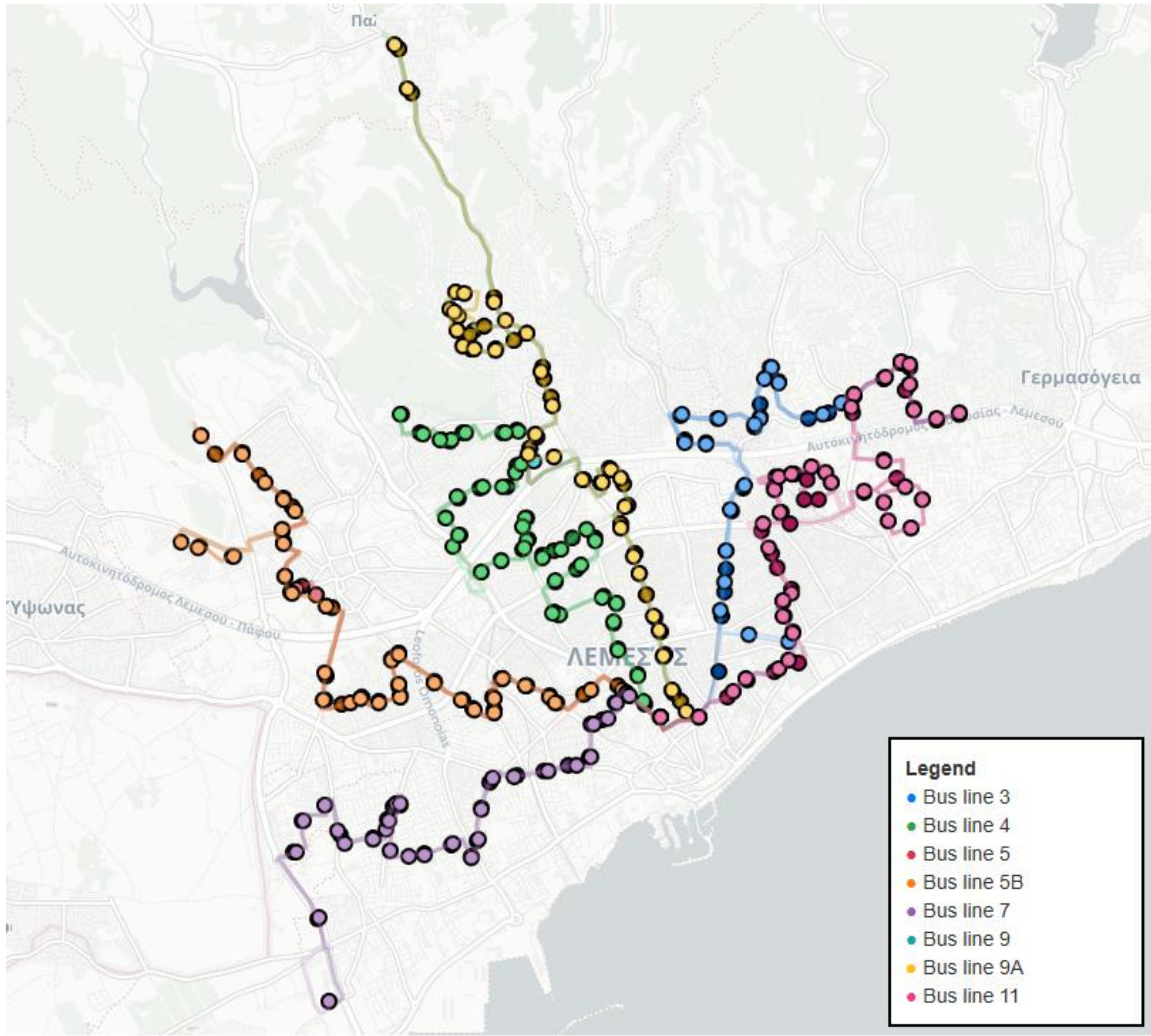

Figure 1. Limassol network of the electric bus lines.

Key Findings

To evaluate the proposed approach, we compared three planning models:

1. the standalone upper-level formulation with fixed *ToU* tariffs *(only the perspective of the electric bus operator)*,
2. the bi-level model solved by **the nested heuristic algorithm**, and
3. the bi-level model solved to **global optimality via the KKT-based MPCC**.

The results show:

- The bi-level model delivers **lower daily charging costs** and **higher renewable energy usage** compared to the standalone upper-level solution.
- The exact MPCC solution achieves the best overall performance, improving both operator cost and energy authority revenue simultaneously.
- The **nested heuristic produces near-optimal results** with a minimal, but considerable, optimality gap, offering a practical alternative for larger instances.
- **The model application showcases that coordinated tariff-setting and charger deployment enhances renewable energy integration without sacrificing operational feasibility.**
- Application to the **Limassol network validates real-world viability** and demonstrates substantial operational and environmental benefits.

	O_1 (\$)	O_2 (\$)	Daily Renewable Energy (kWh)	Daily charging costs
Upper-level	628,878.92	194.57	972.86	9.59
Nested algorithm	622,585.51	175.79	979.26	6.15
Exact solution via KKT	621.606.66	196.01	980.09	5.61

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